

Module 4

Doubly Reinforced Beams – Theory and Problems

Lesson 9 Doubly Reinforced Beams – Theory

Instructional Objectives:

At the end of this lesson, the student should be able to:

- design the amounts of compression and tensile reinforcement if the b , d , d' , f_{ck} , f_y and M_u are given, and
- determine the moment of resistance of a beam if b , d , d' , f_{ck} , f_y , A_{sc} and A_{st} are given.

4.9.1 Introduction

This lesson illustrates the application of the theory of doubly reinforced beams in solving the two types of problems mentioned in Lesson 8. Both the design and analysis types of problems are solved by (i) direct computation method, and (ii) using tables of SP-16. The step by step solution of the problems will help in understanding the theory of Lesson 8 and its application.

4.9.2 Numerical problems

4.9.2.1 Problem 4.1

Design a simply supported beam of effective span 8 m subjected to imposed loads of 35 kN/m. The beam dimensions and other data are: $b = 300$ mm, $D = 700$ mm, M 20 concrete, Fe 415 steel (Fig. 4.9.1). Determine f_{sc} from d'/d as given in Table 4.2 of Lesson 8.

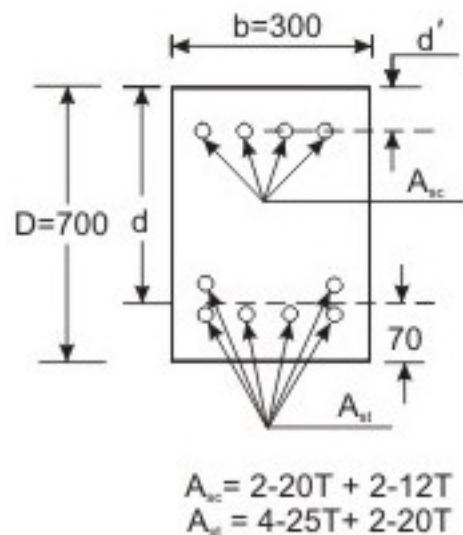


Fig. 4.9.1: Problem 4.1

(a) Solution by direct computation method

Dead load of the beam = $0.3 (0.7) (25) = 5.25$ kN/m

Imposed loads (given) = 35.00 kN/m

Total loads = $5.25 + 35.00 = 40.25$ kN/m

$$\text{Factored bending moment} = (1.5) \frac{wl^2}{8} = \frac{(1.5)(40.25)(8)(8)}{8} = 482.96 \text{ kNm}$$

Assuming $d' = 70$ mm, $d = 700 - 70 = 630$ mm

$$\frac{x_{u, \max}}{d} = 0.48 \text{ gives } x_{u, \max} = 0.48 (630) = 302.4 \text{ mm}$$

Step 1: Determination of $M_{u, \lim}$ and $A_{st, \lim}$

$$\begin{aligned} M_{u, \lim} &= 0.36 \left(\frac{x_{u, \max}}{d} \right) \left(1 - 0.42 \frac{x_{u, \max}}{d} \right) b d^2 f_{ck} \\ (4.2) \quad &= 0.36(0.48) \{ 1 - 0.42 (0.48) \} (300) (630)^2 (20) (10^{-6}) \text{ kNm} \\ &= 328.55 \text{ kNm} \end{aligned}$$

$$A_{st, \lim} = \frac{M_{u, \lim}}{0.87 f_y (d - 0.42 x_{u, \max})} \quad (6.8)$$

$$\text{So, } A_{st1} = \frac{328.55 (10^6) \text{ Nmm}}{0.87 (415) \{ 630 - 0.42 (0.48) 630 \}} = 1809.14 \text{ mm}^2$$

Step 2: Determination of M_{u2} , A_{sc} , A_{st2} and A_{st}

(Please refer to Eqs. 4.1, 4.4, 4.6 and 4.7 of Lesson 8.)

$$M_{u2} = M_u - M_{u, \lim} = 482.96 - 328.55 = 154.41 \text{ kNm}$$

Here, $d'/d = 70/630 = 0.11$

From Table 4.2 of Lesson 8, by linear interpolation, we get,

$$f_{sc} = 353 - \frac{353 - 342}{5} = 350.8 \text{ N/mm}^2$$

$$A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc})(d - d')} = \frac{154.41(10^6) \text{ Nmm}}{\{350.8 - 0.446(20)\}(630 - 70) \text{ N/mm}} = 806.517 \text{ mm}^2$$

$$A_{st2} = \frac{A_{sc}(f_{sc} - f_{cc})}{0.87 f_y} = \frac{806.517(350.8 - 8.92)}{(0.87)(415)} = 763.694 \text{ mm}^2$$

$$A_{st} = A_{st1} + A_{st2} = 1809.14 + 763.621 = 2572.834 \text{ mm}^2$$

Step 3: Check for minimum and maximum tension and compression steel.

(vide sec.4.8.5 of Lesson 8)

(i) In compression:

$$(a) \text{ Minimum } A_{sc} = \frac{0.2}{100}(300)(700) = 420 \text{ mm}^2$$

$$(b) \text{ Maximum } A_{sc} = \frac{4}{100}(300)(700) = 8400 \text{ mm}^2$$

Thus, $420 \text{ mm}^2 < 806.517 \text{ mm}^2 < 8400 \text{ mm}^2$. Hence, o.k.

(ii) In tension:

$$(a) \text{ Minimum } A_{st} = \frac{0.85 b d}{f_y} = \frac{0.85(300)(630)}{415} = 387.1 \text{ mm}^2$$

$$(b) \text{ Maximum } A_{st} = \frac{4}{100}(300)(700) = 8400 \text{ mm}^2$$

Here, $387.1 \text{ mm}^2 < 2572.834 \text{ mm}^2 < 8400 \text{ mm}^2$. Hence, o.k.

Step 4: Selection of bar diameter and numbers.

(i) for A_{sc} : Provide 2-20 T + 2-12 T (= 628 + 226 = 854 mm^2)

(ii) for A_{st} : Provide 4-25 T + 2-20 T (= 1963 + 628 = 2591 mm^2)

It may be noted that A_{st} is provided in two layers in order to provide adequate space for concreting around reinforcement. Also the centroid of the tensile bars is at 70 mm from bottom (Fig. 4.9.1).

(b) Solution by use of table of SP-16

For this problem, $\frac{M_u}{b d^2} = \frac{482.96(10^6)}{300(630)^2} = 4.056$ and $d'/d = \frac{70}{630} = 0.11$.

Table 50 of SP-16 gives ρ_t and ρ_c for $\frac{M_u}{b d^2} = 4$ and 4.1 and $d'/d = 0.1$ and 0.15.

The required ρ_t and ρ_c are determined by linear interpolation. The values are presented in Table 4.3 to get the final ρ_t and ρ_c of this problem.

Table 4.3 Calculation of ρ_t and ρ_c

| $\frac{M_u}{b d^2}$ | | $d'/d = 0.1$ | $d'/d = 0.15$ | $d'/d = 0.11$ |
|---------------------|----------|---------------------|---------------|---|
| 4.0 | ρ_t | 1.337 | 1.360 | $1.337 + \frac{0.023(0.01)}{0.05} = 1.342$ |
| | ρ_c | 0.401 | 0.437 | $0.433 + \frac{0.036(0.01)}{0.05} = 0.408$ |
| 4.1 | ρ_t | 1.368 | 1.392 | $1.368 + \frac{0.024(0.01)}{0.05} = 1.373$ |
| | ρ_c | 0.433 | 0.472 | $0.433 + \frac{0.039(0.01)}{0.05} = 0.441$ |
| 4.056 | ρ_t | Not Applicable (NA) | NA | $1.342 + \frac{0.031(0.056)}{0.1} = 1.3594$ |
| | ρ_c | NA | NA | $0.408 + \frac{0.033(0.056)}{0.1} = 0.426$ |

So, $A_{st} = \frac{1.3594(300)(630)}{100} = 2569.26 \text{ mm}^2$

and $A_{sc} = \frac{0.426(300)(630)}{100} = 805.14 \text{ mm}^2$

These values are close to those obtained by direct computation method where $A_{st} = 2572.834 \text{ mm}^2$ and $A_{sc} = 806.517 \text{ mm}^2$. Thus, by using table of SP-16 we

get the reinforcement very close to that of direct computation method. Hence, provide

(i) for A_{sc} : 2-20 T + 2-12 T (= 628 + 226 = 854 mm²)

(ii) for A_{st} : 4-25 T + 2-20 T (= 1963 + 628 = 2591 mm²)

4.9.2.2 Problem 4.2

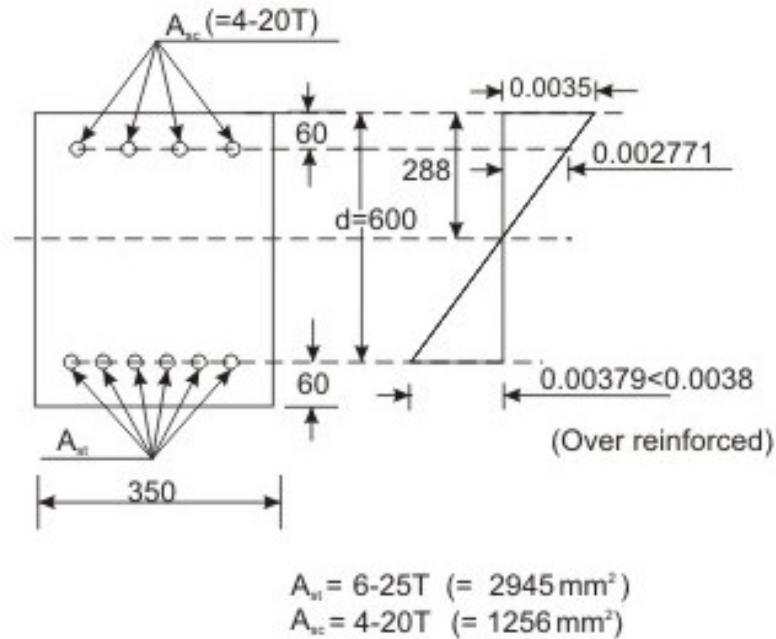


Fig. 4.9.2: Problem 4.2

Determine the ultimate moment capacity of the doubly reinforced beam of $b = 350$ mm, $d' = 60$ mm, $d = 600$ mm, $A_{st} = 2945$ mm² (6-25 T), $A_{sc} = 1256$ mm² (4-20 T), using M 20 and Fe 415 (Fig.4.9.2). Use direct computation method only.

Solution by direct computation method

Step 1: To check if the beam is under-reinforced or over-reinforced.

$$x_{u,\max} = 0.48(600) = 288 \text{ mm}$$

$$\epsilon_{st} = \frac{\epsilon_c (d - x_{u,\max})}{x_{u,\max}} = \frac{0.0035 (600 - 288)}{288} = 0.00379$$

$$\begin{aligned} \text{Yield strain of Fe 415} &= \frac{f_y}{1.15(E_s)} + 0.002 = \frac{415}{(1.15)(2)(10^5)} + 0.002 \\ &= 0.0038 > 0.00379. \end{aligned}$$

Hence, the beam is over-reinforced.

Step 2: To determine $M_{u,lim}$ and $A_{st,lim}$
(vide Eq. 4.2 of Lesson 8 and Table 3.1 of Lesson 5)

$$\begin{aligned} M_{u,lim} &= 0.36 \left(\frac{x_{u,max}}{d} \right) \left(1 - 0.42 \frac{x_{u,max}}{d} \right) b d^2 f_{ck} \\ &= 0.36 (0.48) \{1 - 0.42 (0.48)\} (350) (600)^2 (20) (10^{-6}) \text{ kNm} \\ &= 347.67 \text{ kNm} \end{aligned}$$

From Table 3.1 of Lesson 5, for $f_{ck} = 20 \text{ N/mm}^2$ and $f_y = 415 \text{ N/mm}^2$,

$$A_{st,lim} = \frac{0.96 (350) (600)}{100} = 2016 \text{ mm}^2$$

Step 3: To determine A_{st2} and A_{sc}
(vide Eqs.4.7 and 4.6 of Lesson 8)

$$A_{st2} = A_{st} - A_{st,lim} = 2945 - 2016 = 929 \text{ mm}^2$$

The required A_{sc} will have the compression force equal to the tensile force as given by 929 mm^2 of A_{st2} .

$$\text{So, } A_{sc} = \frac{A_{st2} (0.87 f_y)}{(f_{sc} - f_{cc})}$$

For f_{sc} let us calculate ε_{sc} : (vide Eq. 4.9 of Lesson 8)

$$\varepsilon_{sc} = \frac{0.0035 (x_{u,max} - d')}{x_{u,max}} = \frac{0.0035 (288 - 60)}{288} = 0.002771$$

Table 4.1 of Lesson 8 gives:

$$f_{sc} = 351.8 + \frac{(360.9 - 351.8) (0.002771 - 0.002760)}{(0.00380 - 0.00276)} = 351.896 \text{ N/mm}^2$$

$$\text{So, } A_{sc} = \frac{929(0.87)(415)}{\{351.89 - 0.446(20)\}} = 977.956 \text{ mm}^2$$

Step 4: To determine M_{u2} , M_u and A_{st}
 (Please refer to Eqs. 4.4 and 4.1 of Lesson 8)

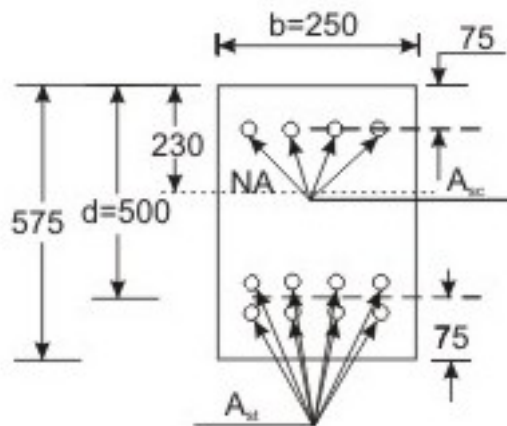
$$\begin{aligned} M_{u2} &= A_{sc} (f_{sc} - f_{cc}) (d - d') \\ &= 977.956 \{351.896 - 0.446(20)\} (600 - 60) (10^{-6}) \text{ kNm} \\ &= 181.12 \text{ kNm} \end{aligned}$$

$$M_u = M_{u, \text{lim}} + M_{u2} = 347.67 + 181.12 = 528.79 \text{ kNm}$$

Therefore, with $A_{st} = A_{st, \text{lim}} + A_{st2} = 2016 + 929 = 2945 \text{ mm}^2$ the required $A_{sc} = 977.956 \text{ mm}^2$ (much less than the provided 1256 mm^2). Hence, o.k.

4.9.3 Practice Questions and Problems with Answers

Q.1: Design a doubly reinforced beam (Fig. 4.9.3) to resist $M_u = 375 \text{ kNm}$ when $b = 250 \text{ mm}$, $d = 500 \text{ mm}$, $d' = 75 \text{ mm}$, $f_{ck} = 30 \text{ N/mm}^2$ and $f_y = 500 \text{ N/mm}^2$, using (i) direct computation method and (ii) using table of SP-16.



$$\begin{aligned} A_{sc} &= 4-16T \quad (= 804 \text{ mm}^2) \\ A_{st} &= 6-20T + 2-12T \quad (= 2110 \text{ mm}^2) \end{aligned}$$

Fig. 4.9.3: Problem of Q 1

A.1: (A) Solution by direct computation method:

From the given data

$$\begin{aligned} M_{u, \text{lim}} &= 0.36 \left(\frac{x_{u, \text{max}}}{d} \right) \left(1 - 0.42 \frac{x_{u, \text{max}}}{d} \right) b d^2 f_{ck} \\ &= 0.36 (0.46) \{ 1 - 0.42 (0.46) \} (250) (500)^2 (30) (10^{-6}) \text{ kNm} \\ &= 250.51 \text{ kNm} \end{aligned}$$

Using the value of $p_t = 1.13$ from Table 3.1 of Lesson 5 for $f_{ck} = 30 \text{ N/mm}^2$ and $f_y = 500 \text{ N/mm}^2$,

$$A_{st, \text{lim}} = \frac{1.13 (250) (500)}{100} = 1412.5 \text{ mm}^2$$

$$M_{u2} = 375 - 250.51 = 124.49 \text{ kNm}$$

From Table 4.2 of Lesson 8, for $d'/d = 75/500 = 0.15$ and $f_y = 500 \text{ N/mm}^2$, we get $f_{sc} = 395 \text{ N/mm}^2$

$$A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc}) (d - d')} = \frac{124.49 (10^6)}{\{ 395 - 0.446 (30) \} (500 - 75)} = 767.56 \text{ mm}^2$$

$$A_{st2} = \frac{A_{sc} (f_{sc} - f_{cc})}{0.87 f_y} = \frac{767.56 \{ 395 - 0.446 (30) \}}{0.87 (500)} = 673.37 \text{ mm}^2$$

$$A_{st} = A_{st, \text{lim}} + A_{st2} = 1412.5 + 673.37 = 2085.87 \text{ mm}^2$$

Alternatively: (use of Table 4.1 of Lesson 8 to determine f_{sc} from ϵ_{sc})

$$x_{u, \text{max}} = 0.46 (500) = 230 \text{ mm}$$

$$\epsilon_{sc} = \frac{0.0035 (230 - 75)}{230} = \frac{0.0035 (155)}{230} = 0.002359$$

From Table 4.1

$$f_{sc} = 391.3 + \frac{(413.0 - 391.3)(0.002359 - 0.00226)}{(0.00277 - 0.00226)} = 395.512 \text{ N/mm}^2$$

$$A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc})(d - d')} = \frac{124.49(10^6)}{\{395.512 - 0.446(30)\}(500 - 75)} = 766.53 \text{ mm}^2$$

$$A_{st2} = \frac{A_{sc}(f_{sc} - f_{cc})}{0.87 f_y} = \frac{766.53(382.132)}{0.87(500)} = 673.369 \text{ mm}^2$$

$$A_{st} = A_{st, \text{lim}} + A_{st2} = 1412.5 + 673.369 = 2085.869 \text{ mm}^2$$

Check for minimum and maximum A_{st} and A_{sc}

$$(i) \text{ Minimum } A_{st} = \frac{0.85 b d}{f_y} = \frac{0.85(250)(500)}{500} = 212.5 \text{ mm}^2$$

$$(ii) \text{ Maximum } A_{st} = 0.04 b D = 0.04(250)(575) = 5750 \text{ mm}^2$$

$$(iii) \text{ Minimum } A_{sc} = \frac{0.2 b D}{100} = \frac{0.2(250)(575)}{100} = 287.5 \text{ mm}^2$$

$$(iv) \text{ Maximum } A_{sc} = 0.04 b D = 0.04(250)(575) = 5750 \text{ mm}^2$$

Hence, the areas of reinforcement satisfy the requirements.

So, provide (i) 6-20 T + 2-12 T = 1885 + 226 = 2111 mm² for A_{st}

(ii) 4-16 T = 804 mm² for A_{sc}

(B) Solution by use of table of SP-16

From the given data, we have

$$\frac{M_u}{b d^2} = \frac{375(10^6)}{250(500)^2} = 6.0$$

$$d'/d = 75/500 = 0.15$$

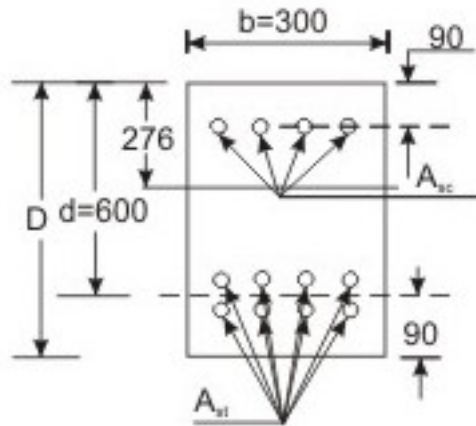
Table 56 of SP-16 gives: $\rho_t = 1.676$ and $\rho_c = 0.619$

$$\text{So, } A_{st} = \frac{(1.676)(250)(500)}{100} = 2095 \text{ mm}^2$$

$$\text{and } A_{sc} = \frac{(0.619)(250)(500)}{100} = 773.75 \text{ mm}^2$$

These values are close to those of (A). Hence, provide 6-20 T + 2-12 T as A_{st} and 4-16 T as A_{sc} .

Q.2: Determine the moment of resistance of the doubly reinforced beam (Fig. 4.9.4) with $b = 300$ mm, $d = 600$ mm, $d' = 90$ mm, $f_{ck} = 30$ N/mm², $f_y = 500$ N/mm², $A_{sc} = 2236$ mm² (2-32 T + 2-20 T), and $A_{st} = 4021$ mm² (4-32 T + 4-16 T). Use (i) direct computation method and (ii) tables of SP-16.



$$A_{sc} = 2-32T + 2-20T (= 2236 \text{ mm}^2)$$

$$A_{st} = 4-32T + 4-16T (= 4021 \text{ mm}^2)$$

Fig. 4.9.4: Problem of Q 2

A.2: (i) Solution by direct computation method:

$$x_{u, max} = 0.46 (600) = 276 \text{ mm}$$

$$\varepsilon_{st} = \frac{0.0035 (600 - 276)}{276} = 0.0041086$$

$\varepsilon_{yield} = 0.00417$. So $\varepsilon_{st} < \varepsilon_{yield}$ i.e. the beam is over-reinforced.

For $d'/d = 0.15$ and $f_y = 500 \text{ N/mm}^2$, Table 4.2 of Lesson 8 gives: $f_{sc} = 395 \text{ N/mm}^2$ and with $f_{ck} = 30 \text{ N/mm}^2$, Table 3.1 of Lesson 5 gives $\rho_{t, lim} = 1.13$.

$$A_{st,lim} = \frac{1.13 (300) (600)}{100} = 2034 \text{ mm}^2$$

$$\begin{aligned} M_{u,lim} &= 0.36 \left(\frac{x_{u,max}}{d} \right) \left(1 - 0.42 \frac{x_{u,max}}{d} \right) b d^2 f_{ck} \\ &= 0.36 (0.46) \{ 1 - 0.42 (0.46) \} (300) (600)^2 (30) (10^{-6}) \text{ kNm} \\ &= 432.88 \text{ kNm} \end{aligned}$$

$$A_{st2} = 4021 - 2034 = 1987 \text{ mm}^2$$

$$(A_{sc})_{required} = \frac{A_{st2} (0.87) f_y}{(f_{sc} - f_{cc})} = \frac{1987 (0.87) (500)}{\{395 - 0.446(30)\}} = 2264.94 \text{ mm}^2 > 2236 \text{ mm}^2$$

So, A_{st2} of 1987 mm^2 is not fully used. Let us determine A_{st2} required when $A_{sc} = 2236 \text{ mm}^2$.

$$A_{st2} = \frac{A_{sc} (f_{sc} - f_{cc})}{0.87 f_y} = \frac{2236 \{395 - 0.446(30)\}}{(0.87) (500)} = 1961.61 \text{ mm}^2$$

$$A_{st} = A_{st, lim} + A_{st2} = 2034 + 1961.61 = 3995.61 \text{ mm}^2 < 4021 \text{ mm}^2.$$

Hence, o.k.

$$\begin{aligned} \text{With } A_{st2} &= 1961.61 \text{ mm}^2, \quad M_{u2} = A_{st2} (0.87 f_y) (d - d') \\ &= 1961.61 (0.87) (500) (600 - 75) (10^{-6}) \text{ kNm} = 447.98268 \text{ kNm} \end{aligned}$$

Again, when $A_{sc} = 2236 \text{ mm}^2$ (as provided)

$$M_{u2} = A_{sc} (f_{sc} - f_{cc}) (d - d')$$

$$= 2236 \{395 - 0.446 (30)\} (600 - 75) (10^{-6}) \text{ kNm} = 447.9837 \text{ kNm}$$

two)

$$M_u = M_{u, lim} + M_{u2} = 432.88 + 447.98 \text{ (} M_{u2} \text{ is taken the lower of the two)}$$

$$= 880.86 \text{ kNm}$$

Hence, the moment of resistance of the beam is 880.86 kNm.

Alternatively f_{sc} can be determined from Table 4.1 of Lesson 8.

Using the following from the above:

$$x_{u, max} = 276 \text{ mm}$$

$$A_{st, lim} = 2034 \text{ mm}^2$$

$$M_{u, lim} = 432.88 \text{ kNm}$$

$$A_{st2} = 1987 \text{ mm}^2$$

To find $(A_{sc})_{required}$

$$\varepsilon_{st} = \frac{0.0035 (276 - 90)}{276} = 0.00236$$

Table 4.1 of Lesson 8 gives:

$$f_{sc} = 391.3 + \frac{(413 - 391.3) (0.00236 - 0.00226)}{(0.00277 - 0.00226)} = 395.55 \text{ N/mm}^2$$

$$(A_{sc})_{required} = \frac{A_{st2} (0.87) f_y}{(f_{sc} - f_{cc})} = \frac{1987 (0.87) (500)}{\{395.55 - 0.446 (30)\}}$$

$$= 2261.68 \text{ mm}^2 > 2236 \text{ mm}^2$$

So, it is not o.k.

Let us determine A_{st2} required when $A_{sc} = 2236 \text{ mm}^2$.

$$A_{st2} = \frac{A_{sc} (f_{sc} - f_{cc})}{0.87 f_y} = \frac{2236 \{395.55 - 0.446(30)\}}{(0.87)(500)} = 1964.44 \text{ mm}^2$$

$$A_{st} = A_{st, lim} + A_{st2} = 2034 + 1964.44 = 3998.44 \text{ mm}^2 < 4021 \text{ mm}^2.$$

So, o.k.

$$\begin{aligned} M_{u2} \text{ (when } A_{st2} = 1964.44 \text{ mm}^2) &= A_{st2} (0.87 f_y) (d - d') \\ &= 1964.44 (0.87) (500) (600 - 75) (10^{-6}) \text{ kNm} \\ &= 448.63 \text{ kNm} \end{aligned}$$

For $A_{sc} = 2236 \text{ mm}^2$,

$$\begin{aligned} M_{u2} &= A_{sc} (f_{sc} - f_{cc}) (d - d') \\ &= 2236 \{(395.55 - 0.446(30))\} (600 - 75) (10^{-6}) \text{ kNm} \\ &= 2236 (382.17) (525) (10^{-6}) \text{ kNm} \\ &= 448.63 \text{ kNm} \end{aligned}$$

Both the M_{u2} values are the same. So,

$$\begin{aligned} M_u &= M_{u, lim} + M_{u2} = 432.88 + 448.63 \\ &= 881.51 \text{ kNm} \end{aligned}$$

Here, the $M_u = 881.51 \text{ kNm}$.

(ii) Solution by using table of SP-16

From the given data:

$$p_t = \frac{4021(100)}{300(600)} = 2.234$$

$$p_c = \frac{2236(100)}{300(600)} = 1.242$$

$$d'/d = 0.15$$

Table 56 of SP-16 is used first considering $d'/d = 0.15$ and $p_t = 2.234$, and secondly, considering $d'/d = 0.15$ and $p_c = 1.242$. The calculated values of p_c and M_u/bd^2 for the first and p_t and M_u/bd^2 for the second cases are presented below separately. Linear interpolation has been done.

(i) When $d'/d = 0.15$ and $p_t = 2.234$

$$\frac{M_u}{bd^2} = 8.00 + \frac{(8.1-8.0)(2.234-2.218)}{(2.245-2.218)} = 8.06$$

$$p_c = 1.235 + \frac{(1.266-1.235)(0.016)}{(0.027)} = 1.253 > 1.242$$

So, this is not possible.

(ii) When $d'/d = 0.15$ and $p_c = 1.242$

$$\frac{M_u}{bd^2} = 8.00 + \frac{(8.1-8.0)(1.242-1.235)}{(1.266-1.235)} = 8.022$$

$$p_t = 2.218 + \frac{(2.245-2.218)(1.242-1.235)}{(1.266-1.235)} = 2.224 < 2.234$$

So, $M_u = 8.022 (300) (600)^2 (10^{-6}) = 866.376 \text{ kNm}$.

Hence, o.k.

4.9.4 References

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4.9.5 Test 9 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

TQ.1: Design a simply supported beam of effective span 8 m subjected to imposed loads of 35 kN/m. The beam dimensions and other data are: $b = 300$ mm, $D = 700$ mm, M 20 concrete, Fe 415 steel (Fig. 4.9.1). Determine f_{sc} from strain ϵ_{sc} as given in Table 4.1 of Lesson 8.

A.TQ.1: This problem is the same as Problem 4.1 in sec. 4.9.2.1 except that here the f_{sc} is to be calculated using Table 4.1 instead of Table 4.2.

Step 1: Here, the Step 1 will remain the same as that of Problem 4.1.

Step 2: Determination of M_{u2} , A_{sc} , A_{st2} and A_{st}

$$M_{u2} = M_u - M_{u, \text{lim}} = 482.96 - 328.55 = 154.41 \text{ kNm}$$

From strain triangle: (Fig. 4.8.2 of Lesson 8)

$$\epsilon_{sc} = \frac{0.0035 (302.4 - 70)}{302.4} = 0.00269$$

$$f_{sc} \text{ (from Table 4.1 of Lesson 8)} = 342.8 + \frac{(351.8 - 342.8)}{(0.00276 - 0.00241)} (0.00269 - 0.00241)$$

$$= 350 \text{ N/mm}^2$$

$$A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc}) (d - d')} = \frac{154.41 (10^6)}{\{350 - 0.446 (20)\} (630 - 70) \text{ N/mm}} = 808.41 \text{ mm}^2$$

$$A_{st2} = \frac{A_{sc} (f_{sc} - f_{cc})}{0.87 f_y} = \frac{808.41 (341.08)}{(0.87) (415)} = 763.696 \text{ mm}^2$$

$$A_{st} = A_{st1} + A_{st2} = 1809.14 + 763.696 = 2572.836 \text{ mm}^2$$

$$A_{sc} = 808.41 \text{ mm}^2$$

Steps 3 & 4 will also remain the same as those of Problem 4.1.

Hence, provide 2-20 T + 2-12 T (854 mm²) as A_{sc} and 4-25 T + 2-20 T (2591 mm²) as A_{st} .

TQ.2: Determine the ultimate moment capacity of the doubly reinforced beam of $b = 350 \text{ mm}$, $d' = 60 \text{ mm}$, $d = 600 \text{ mm}$, $A_{st} = 2945 \text{ mm}^2$ (6-25 T), $A_{sc} = 1256 \text{ mm}^2$ (4-20 T), using M 20 and Fe 415 (Fig. 4.9.2). Use table of SP-16 only.

A.TQ.2: Solution by using table of SP-16

This problem is the same as that of Problem 4.2 of sec. 4.9.2.2, which has been solved by direct computation method. Here, the same is to be solved by using SP-16.

The needed parameters are:

$$d'/d = 60/600 = 0.1$$

$$p_t = \frac{A_{st} (100)}{b d} = \frac{2945 (100)}{350(600)} = 1.402$$

$$p_c = \frac{A_{sc} (100)}{b d} = \frac{1256 (100)}{350 (600)} = 0.5981$$

Here, we need to use Table 50 for $f_{ck} = 20 \text{ N/mm}^2$ and $f_y = 415 \text{ N/mm}^2$. The table gives values of M_u/bd^2 for (i) d'/d and p_t and (ii) d'/d and p_c . So, we will consider both the possibilities and determine M_u .

(i) Considering Table 50 of SP-16 when $d'/d = 0.1$ and $p_t = 1.402$:

Interpolating the values of M_u/bd^2 at $p_t = 1.399$ and 1.429 , we get

$$\left(\frac{M_u}{b d^2} \right)_{p_t=1.402} = 4.2 + \frac{(4.3 - 4.2)(1.402 - 1.399)}{(1.429 - 1.399)} = 4.21$$

$$\text{the corresponding } (p_c)_{p_t=1.402} = 0.466 + \frac{(0.498 - 0.466)(1.402 - 1.399)}{(1.429 - 1.399)} = 0.4692$$

But, p_c provided is 0.5981 indicates that extra compression reinforcement has been used.

So, we get

$$M_u = 4.21 b d^2 = (4.21) (350) (600)^2 (10^{-6}) = 530.46 \text{ kNm} \quad \text{when } A_{st} = 2945 \text{ mm}^2 \text{ and } A_{sc} = 985.32 \text{ mm}^2, \text{ i.e. } 270.69 \text{ mm}^2 (= 1256 - 985.32) \text{ of compression steel is extra.}$$

(ii) Considering $d'/d = 0.1$ and $p_c = 0.5981$, we get by linear interpolation

$$\left(\frac{M_u}{b d^2} \right)_{p_c=0.5981} = 4.6 + \frac{(4.7 - 4.6)(0.5981 - 0.595)}{(0.628 - 0.595)} = 4.61$$

the corresponding p_t is:

$$(p_t)_{p_c=0.5981} = 1.522 + \frac{(1.533 - 1.522)(0.5981 - 0.595)}{(0.628 - 0.595)} = 1.5231$$

The provided $p_t = 1.402$ indicates that the tension steel is insufficient by 254.31 mm^2 as shown below:

Amount of additional A_{st} still required =

$$\frac{(1.5231 - 1.402) (350) (600)}{100} = 254.31 \text{ mm}^2$$

If this additional steel is provided, then the M_u of this beam becomes:

$$M_u = 4.61 b d^2 = 4.61 (350) (600)^2 (10^{-6}) \text{ kNm} = 580.86 \text{ kNm}$$

The above two results show that the moment of resistance of this beam is the lower of the two. So, $M_u = 530.46 \text{ kNm}$. By direct computation the $M_u = 528.79 \text{ kNm}$. The two results are in good agreement.

4.9.6 Summary of this Lesson

This lesson presents solutions of four numerical problems covering both design and analysis types. These problems are solved by two methods: (i) direct computation method and (ii) using table of SP-16. Two problems are illustrated in the lesson and the other two are given in the practice problem and test of this lesson. The solutions will help in understanding the step by step application of the theory of doubly reinforced beams given in Lesson 8.